The shortening of the neutral surface of the beam is given by Eq. 14 to be

$$\begin{aligned} \Delta l &= \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx \\ &= \frac{1}{2}\int_0^{70}\left[-0.00632 + 2.083(10^{-6})x^2\right]^2 dx \\ &\qquad + \frac{1}{2}\int_{70}^{100} [-0.04033 + 9.716(10^{-4})x - 4.857(10^{-6})x^2]^2\, dx \end{aligned}$$

$$\text{or} \qquad \Delta l = 0.00244 \text{ in} \qquad \text{(a shortening)}$$

In addition to the shortening of the neutral surface, the lower edge of the left end moves to the left by an amount $\theta_A c$ or $0.00632(3) = 0.01896$ in. Similarly, the lower edge of the right end moves to the right by an amount $\theta_B c$ or $0.00828(3) = 0.02484$ in. Evaluating the motion of the lower edges in this manner is equivalent to solving Eq. 15 for the total strain in the lower fibers of the beam.

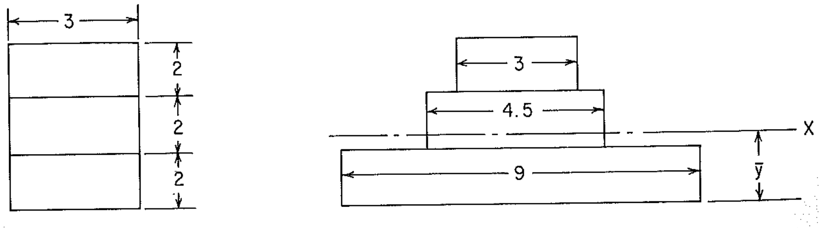
The total relative motion of the lower edges of the end sections is therefore a moving apart by an amount $0.01896 + 0.02484 - 0.00244 = 0.04136$ in.

7.2 *Composite beams and bimetallic strips*

Beams that are constructed of more than one material can be treated by using an equivalent width technique if the maximum stresses in each of the several materials remain within the proportional limit. An equivalent cross section is developed in which the width of each component parallel to the principal axis of bending is increased in the same proportion that the modulus of elasticity of that component makes with the modulus of the assumed material of the equivalent beam.

EXAMPLE

The beam cross section shown in Fig. 7.4*a* is composed of three portions of equal width and depth. The top portion is made of aluminum for which $E_A = 10 \cdot 10^6$ lb/in²; the center is made of brass for which $E_B = 15 \cdot 10^6$ lb/in²; and the bottom is made of steel for which $E_S = 30 \cdot 10^6$ lb/in². Figure 7.4*b* shows the equivalent cross section, which is assumed to be made of aluminum. For this equivalent cross section the centroid must be located and the moment of inertia determined for the centroidal axis.



(a) (b)

Fig. 7.4

given by Eq. 14 to be

$$083(10^{-6})x^2\Big]^2\,dx$$

$$3 + 9.716(10^{-4})x - 4.857(10^{-6})x^2]^2\,dx$$

shortening)

lower edge of the left end moves to
Similarly, the lower edge of the right
= 0.02484 in. Evaluating the motion
ng Eq. 15 for the total strain in the

l sections is therefore a moving apart
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of three portions of equal width and
$= 10 \cdot 10^6$ lb/in^2; the center is made
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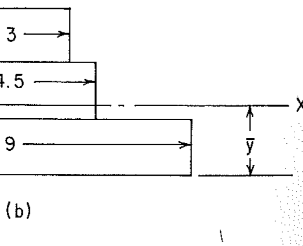

Solution

$$\bar{y} = \frac{3(2)5 + 4.5(2)(3) + 9(2)(1)}{6 + 9 + 18} = 2.27 \text{ in}$$

$$I_x = \frac{3(2^3)}{12} + 6(5 - 2.27)^2 + \frac{4.5(2^3)}{12} + 9(3 - 2.27)^2 + \frac{9(2^3)}{12} + 18(2.27 - 1)^2$$
$$= 89.5 \text{ in}^4$$

The equivalent stiffness EI of this beam is therefore $10 \cdot 10^6(89.5)$, or $895 \cdot 10^6$ lb in^2.

A flexure stress computed by $\sigma = Mc/I_x$ will give a stress in the equivalent beam which can thereby be converted into the stress in the actual composite beam by multiplying by the modulus ratio. If a bending moment of 300,000 in-lb were applied to a beam with the cross section shown, the stress at the top surface on the equivalent beam would be $\sigma = 300{,}000(6 - 2.27)/89.5$, or 12,500 lb/in^2. Since the material at the top is the same in both the actual and equivalent beams, this is also the maximum stress in the aluminum portion of the actual beam. The stress at the bottom of the equivalent beam would be $\sigma = 300{,}000(2.27)/89.5 = 7{,}620$ lb/in^2. The actual stress at the bottom of the steel portion of the beam would be $\sigma = 7{,}620(30)/10 = 22{,}900$ lb/in^2.

Bimetallic strips are widely used in instruments to sense or control temperatures. The following formula gives the equivalent properties of the strip for which the cross section is shown in Fig. 7.5:

$$\text{Equivalent } EI = \frac{wt_b^3 t_a E_b E_a}{12(t_a E_a + t_b E_b)} K_1$$

$$\text{where } K_1 = 4 + 6\frac{t_a}{t_b} + 4\left(\frac{t_a}{t_b}\right)^2 + \frac{E_a}{E_b}\left(\frac{t_a}{t_b}\right)^3 + \frac{E_b}{E_a}\frac{t_b}{t_a}$$

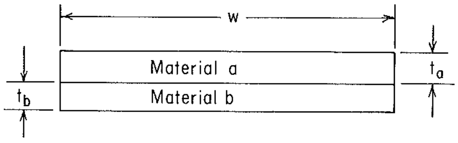

Fig. 7.5

All of the formulas in Table 3, cases 1 to 5, can be applied to the bimetallic beam by using this equivalent value of EI. Since a bimetallic strip is designed to deform when its temperature differs from T_o, the temperature at which the strip is straight, Table 3, case 6, can be used to solve for reaction forces and moments as well as deformations of the bimetallic strip under a uniform temperature T. To do this, the term $\gamma(T_2 - T_1)/t$ is replaced by the term $6(\gamma_b - \gamma_a)(T - T_o)(t_a + t_b)/(t_b^2 K_1)$ and EI is replaced by the equivalent EI given above.

After the moments and deformations have been determined, the flexure stresses can be computed. The stresses due to the bending moments caused by the restraints and any applied loads are given by the following expressions: